SATPREP Name___________________________________

Assignment Mathmatical Induction (Divisibility) Date________________

Use mathematical induction to prove that each statement is true for all positive integers.

1) $4n^3 + 8n$ is divisible by 6

2) $11^n - 4^n$ is divisible by 7

3) $12n^2 + 20n$ is divisible by 8

4) $19n^2 + 7n$ is divisible by 2

5) $7n^2 - 5n$ is divisible by 2

6) $6n^2 + 2n$ is divisible by 4

7) $9^n - 2^n$ is divisible by 7

8) $11^n - 3^n$ is divisible by 8

9) 4 is a factor of $5^n + 3$

10) $11^n - 2^n$ is divisible by 9

Answers to Assignment Mathmatical Induction (Divisibility)

1) Let P_n be the statement $4n^3 + 8n$ is divisible by 6

Anchor Step

P_1 is true: $4 \cdot 1^3 + 8$ is divisible by 6

Inductive Hypothesis

Assume that $4k^3 + 8k$ is divisible by 6. Therefore, $4k^3 + 8k = 6r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $4(k + 1)^3 + 8(k + 1)$ is divisible by 6

$4(k^3 + 3k^2 + 3k + 1) + 8k + 8$

$4k^3 + 12k^2 + 12k + 4 + 8k + 8$

$4k^3 + 8k + 12k^2 + 12k + 12$

$6r + 12k^2 + 12k + 12$

$6(r + 2k^2 + 2k + 2)$

Conclusion

By induction P_n is true for all $n \geq 1$.

2) Let P_n be the statement $11^n - 4^n$ is divisible by 7

Anchor Step

P_1 is true: $11^1 - 4^1$ is divisible by 7

Inductive Hypothesis

Assume that $11^k - 4^k$ is divisible by 7. Therefore, $11^k - 4^k = 7r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $11^{(k+1)} - 4^{(k+1)}$ is divisible by 7

$11 \cdot 11^k - 4 \cdot 4^k$

$7 \cdot 11^k + 11^k + 11^k + 11^k + 11^k - 4^k - 4^k - 4^k - 4^k$

$7 \cdot 11^k + 11^k - 4^k + 11^k - 4^k + 11^k - 4^k + 11^k - 4^k$

$7 \cdot 11^k + 7r + 7r + 7r + 7r$

$7(11^k + r + r + r + r)$

Conclusion

By induction P_n is true for all $n \geq 1$.

3) Let P_n be the statement $12n^2 + 20n$ is divisible by 8

Anchor Step

P_1 is true: $12 \cdot 1^2 + 20$ is divisible by 8

Inductive Hypothesis

Assume that $12k^2 + 20k$ is divisible by 8. Therefore, $12k^2 + 20k = 8r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $12(k + 1)^2 + 20(k + 1)$ is divisible by 8

$12k^2 + 24k + 12 + 20k + 20$

$12k^2 + 20k + 24k + 32$

$8r + 24k + 32$

$8(r + 3k + 4)$

Conclusion

By induction P_n is true for all $n \geq 1$.

4) Let P_n be the statement $19n^2 + 7n$ is divisible by 2

Anchor Step

P_1 is true: $19 \cdot 1^2 + 7$ is divisible by 2

Inductive Hypothesis

Assume that $19k^2 + 7k$ is divisible by 2. Therefore, $19k^2 + 7k = 2r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $19(k+1)^2 + 7(k+1)$ is divisible by 2

$19k^2 + 38k + 19 + 7k + 7$

$19k^2 + 7k + 38k + 26$

$2r + 38k + 26$

$2(r + 19k + 13)$

Conclusion

By induction P_n is true for all $n \geq 1$.

5) Let P_n be the statement $7n^2 - 5n$ is divisible by 2

Anchor Step

P_1 is true: $7 \cdot 1^2 - 5$ is divisible by 2

Inductive Hypothesis

Assume that $7k^2 - 5k$ is divisible by 2. Therefore, $7k^2 - 5k = 2r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $7(k+1)^2 - 5(k+1)$ is divisible by 2

$7k^2 + 14k + 7 - 5k - 5$

$7k^2 - 5k + 14k + 2$

$2r + 14k + 2$

$2(r + 7k + 1)$

Conclusion

By induction P_n is true for all $n \geq 1$.

6) Let P_n be the statement $6n^2 + 2n$ is divisible by 4

Anchor Step

P_1 is true: $6 \cdot 1^2 + 2$ is divisible by 4

Inductive Hypothesis

Assume that $6k^2 + 2k$ is divisible by 4. Therefore, $6k^2 + 2k = 4r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $6(k+1)^2 + 2(k+1)$ is divisible by 4

$6k^2 + 12k + 6 + 2k + 2$

$6k^2 + 2k + 12k + 8$

$4r + 12k + 8$

$4(r + 3k + 2)$

Conclusion

By induction P_n is true for all $n \geq 1$.

7) Let P_n be the statement $9^n - 2^n$ is divisible by 7

Anchor Step

P_1 is true: $9^1 - 2^1$ is divisible by 7

Inductive Hypothesis

Assume that $9^k - 2^k$ is divisible by 7. Therefore, $9^k - 2^k = 7r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $9^{(k+1)} - 2^{(k+1)}$ is divisible by 7

$9 \cdot 9^k - 2 \cdot 2^k$

$7 \cdot 9^k + 9^k + 9^k - 2^k - 2^k$

$7 \cdot 9^k + 9^k - 2^k + 9^k - 2^k$

$7 \cdot 9^k + 7r + 7r$

$7\left(9^k + r + r\right)$

Conclusion

By induction P_n is true for all $n \geq 1$.

8) Let P_n be the statement $11^n - 3^n$ is divisible by 8

Anchor Step

P_1 is true: $11^1 - 3^1$ is divisible by 8

Inductive Hypothesis

Assume that $11^k - 3^k$ is divisible by 8. Therefore, $11^k - 3^k = 8r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $11^{(k+1)} - 3^{(k+1)}$ is divisible by 8

$11 \cdot 11^k - 3 \cdot 3^k$

$8 \cdot 11^k + 11^k + 11^k + 11^k - 3^k - 3^k - 3^k$

$8 \cdot 11^k + 11^k - 3^k + 11^k - 3^k + 11^k - 3^k$

$8 \cdot 11^k + 8r + 8r + 8r$

$8\left(11^k + r + r + r\right)$

Conclusion

By induction P_n is true for all $n \geq 1$.

9) Let P_n be the statement 4 is a factor of $5^n + 3$

Anchor Step

P_1 is true: 4 is a factor of $5^1 + 3$

Inductive Hypothesis

Assume that 4 is a factor of $5^k + 3$. Therefore, $5^k + 3 = 4r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: 4 is a factor of $5^{(k+1)} + 3$

$5 \cdot 5^k + 3$

$(4 + 1) \cdot 5^k + 3$

$4 \cdot 5^k + 5^k + 3$

$4 \cdot 5^k + 4r$

$4\left(5^k + r\right)$

Conclusion

By induction P_n is true for all $n \geq 1$.

10) Let P_n be the statement $11^n - 2^n$ is divisible by 9

Anchor Step

P_1 is true: $11^1 - 2^1$ is divisible by 9

Inductive Hypothesis

Assume that $11^k - 2^k$ is divisible by 9. Therefore, $11^k - 2^k = 9r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $11^{(k+1)} - 2^{(k+1)}$ is divisible by 9

$$11 \cdot 11^k - 2 \cdot 2^k$$
$$9 \cdot 11^k + 11^k + 11^k - 2^k - 2^k$$
$$9 \cdot 11^k + 11^k - 2^k + 11^k - 2^k$$
$$9 \cdot 11^k + 9r + 9r$$
$$9\left(11^k + r + r\right)$$

Conclusion

By induction P_n is true for all $n \geq 1$.