SATPREP
Assignment: Algebra

1. When the function $f(x) = 6x^4 + 11x^3 - 22x^2 + ax + 6$ is divided by $(x + 1)$ the remainder is -20. Find the value of a.

2. Find the coefficient of x^7 in the expansion of $(2 + 3x)^{10}$, giving your answer as a whole number.

3. The second term of an arithmetic sequence is 7. The sum of the first four terms of the arithmetic sequence is 12. Find the first term, a, and the common difference, d, of the sequence.

4. The sum of the first n terms of an arithmetic sequence is $S_n = 3n^2 - 2n$. Find the nth term u_n.

5. Let $z = x + y\text{i}$. Find the values of x and y if $(1 - \text{i})z = 1 - 3\text{i}$.

6. Let $z_1 = a\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right)$ and $z_2 = b\left(\cos\frac{\pi}{3} + \text{i}\sin\frac{\pi}{3}\right)$.

Express $\left(\frac{z_1}{z_2}\right)^3$ in the form $z = x + y\text{i}$.

7. Express $\frac{3x - 4}{x^2 - x}$ in partial fractions.

8. Given $f(x) = x^2 + x(2 - k) + k^2$, find the range of values of k for which $f(x) > 0$ for all real values of x.

9. Let $z_1 = \frac{\sqrt{6} - \text{i}\sqrt{2}}{2}$, and $z_2 = 1 - \text{i}$.

(a) Write z_1 and z_2 in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r > 0$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

(b) Show that $\frac{z_1}{z_2} = \cos\frac{\pi}{12} + \text{i}\sin\frac{\pi}{12}$.

(c) Find the value of $\frac{z_1}{z_2}$ in the form $a + b\text{i}$, where a and b are to be determined exactly in radical (surd) form. Hence or otherwise find the exact values of $\cos\frac{\pi}{12}$ and $\sin\frac{\pi}{12}$.

10. Find the largest domain for the function $f : x \mapsto \frac{1}{\sqrt{4 - 9x^2}}$.

11. An arithmetic sequence has 5 and 13 as its first two terms respectively.

(a) Write down, in terms of n, an expression for the nth term, a_n.

(b) Find the number of terms of the sequence which are less than 400.

12. The roots α and β of the quadratic equation

$$x^2 - kx + (k + 1) = 0$$

are such that $\alpha^2 + \beta^2 = 13$. Find the possible values of the real number k.