SATPREP
Assignment: Complex Number

1. (a) Express the complex number 8i in polar form.

(b) The cube root of 8i which lies in the first quadrant is denoted by z. Express z

(i) in polar form;

(ii) in cartesian form.

2. Solve, for x, the equation $\log_2 (5x^2 - x - 2) = 2 + 2\log_2 x$.

3. Consider the equation $(1 + 2k)x^2 - 10x + k - 2 = 0$, $k \in \mathbb{R}$. Find the set of values of k for which the equation has real roots.

4. The first four terms of an arithmetic sequence are 2, $a - b$, $2a + b + 7$, and a – 3b, where a and b are constants. Find a and b.

5. Given that $z \in \mathbb{C}$, solve the equation $z^3 - 8\text{i} = 0$, giving your answers in the form $z = r(\cos\theta + \text{i}\sin\theta)$.

6. The polynomial $x^3 + ax^2 - 3x + b$ is divisible by $(x - 2)$ and has a remainder 6 when divided by $(x + 1)$. Find the value of a and of b.

7. Find the values of x for which $|5 - 3x| \le |x + 1|$.

8. Consider the complex number $z = \dfrac{\left(\cos\frac{\pi}{4} - \text{i}\sin\frac{\pi}{4}\right)^2 \left(\cos\frac{\pi}{3} + \text{i}\sin\frac{\pi}{3}\right)^3}{\left(\cos\frac{\pi}{24} - \text{i}\sin\frac{\pi}{24}\right)^4}$.

(a) (i) Find the modulus of z.
(ii) Find the argument of z, giving your answer in radians.

(b) Using De Moivre's theorem, show that z is a cube root of one, *ie* $z = \sqrt[3]{1}$.

(c) Simplify $(1 + 2z)(2 + z^2)$, expressing your answer in the form $a + b\text{i}$, where a and b are **exact** real numbers.

9. Find the values of a and b, where a and b are real, given that $(a + b\text{i})(2 - \text{i}) = 5 - \text{i}$.

10. $z_1 = \left(1+\mathrm{i}\sqrt{3}\right)^m$ and $z_2 = \left(1-\mathrm{i}\right)^n$.

(a) Find the modulus and argument of z_1 and z_2 in terms of m and n, respectively.

(b) **Hence**, find the smallest positive integers m and n such that $z_1 = z_2$.

11. A complex number z is such that $|z| = |z - 3\mathrm{i}|$.

(a) Show that the imaginary part of z is $\frac{3}{2}$.

(b) Let z_1 and z_2 be the two possible values of z, such that $|z| = 3$.

(i) Sketch a diagram to show the points which represent z_1 and z_2 in the complex plane, where z_1 is in the first quadrant.

(ii) Show that $\arg z_1 = \frac{\pi}{6}$.

(iii) Find $\arg z_2$.

(c) Given that $\arg\left(\frac{z_1^k z_2}{2\mathrm{i}}\right) = \pi$, find a value of k.

12. The three terms a, 1, b are in arithmetic progression. The three terms 1, a, b are in geometric progression. Find the value of a and of b given that $a \neq b$.

13. The complex number z satisfies the equation

$$\sqrt{z} = \frac{2}{1-\mathrm{i}} + 1 - 4\mathrm{i}.$$

Express z in the form $x + \mathrm{i}y$ where $x, y \in \mathbb{Z}$.