SATPREP
Assignment : Complex roots of polynomial

1. Let $P(z) = z^3 + az^2 + bz + c$, where a, b, and $c \in \mathbb{R}$. Two of the roots of $P(z) = 0$ are -2 and $(-3 + 2\text{i})$. Find the value of a, of b and of c.

2. $(z + 2\text{i})$ is a factor of $2z^3 - 3z^2 + 8z - 12$. Find the other two factors.

3. The polynomial $\text{P}(z) = z^3 + mz^2 + nz - 8$ is divisible by $(z + 1 + \text{i})$, where $z \in \mathbb{C}$ and $m, n \in \mathbb{R}$. Find the value of m and of n.

4. (a) Express the complex number $1 + \text{i}$ in the form $\sqrt{a}\text{e}^{\text{i}\frac{\pi}{b}}$, where $a, b \in \mathbb{Z}^+$.

(b) Using the result from (a), show that $\left(\frac{1+\text{i}}{\sqrt{2}}\right)^n$, where $n \in \mathbb{Z}$, has only eight distinct values.

(c) **Hence** solve the equation $z^8 - 1 = 0$.

5. Consider the polynomial $p(x) = x^4 + ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbb{R}$.
Given that $1 + \text{i}$ and $1 - 2\text{i}$ are zeros of $p(x)$, find the values of a, b, c and d.

6. Given that $2 + \text{i}$ is a root of the equation $x^3 - 6x^2 + 13x - 10 = 0$ find the other two roots.

7. Given that $z_1 = 2$ and $z_2 = 1 + \text{i}\sqrt{3}$ are roots of the cubic equation $z^3 + bz^2 + cz + d = 0$ where $b, c, d \in \mathbb{R}$,

(a) write down the third root, z_3, of the equation;

(b) find the values of b, c and d;

(c) write z_2 and z_3 in the form $r\text{e}^{\text{i}\theta}$.

8. Consider the equation $z^3 + az^2 + bz + c = 0$, where $a, b, c \in \mathbb{R}$. The points in the Argand diagram representing the three roots of the equation form the vertices of a triangle whose area is 9. Given that one root is $-1 + 3\text{i}$, find

(a) the other two roots;

(b) a, b and c.