SATPREP

Assignment: Trigonometry

1. Let $f(x) = \sqrt{3}e^{2x} \sin x + e^{2x} \cos x$, for $0 \le x \le \pi$. Given that $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$, solve the equation $f(x) = 0$.

2. Solve $\cos 2x - 3 \cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \le x \le 2\pi$.

3. A spring is suspended from the ceiling. It is pulled down and released, and then oscillates up and down. Its length, l centimetres, is modelled by the function $l = 33 + 5\cos((720t)°)$, where t is time in seconds after release.

(a) Find the length of the spring after 1 second.

(b) Find the minimum length of the spring.

(c) Find the first time at which the length is 33 cm.

(d) What is the period of the motion?

4. The following diagram shows triangle ABC.

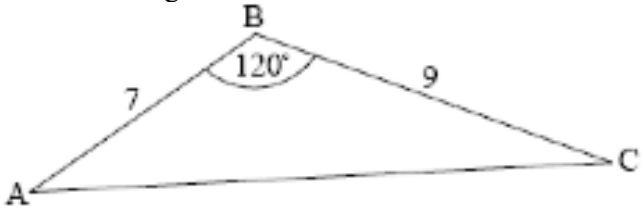


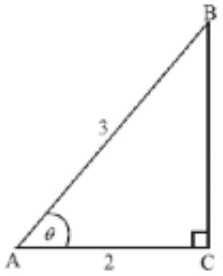
diagram not to scale

AB = 7 cm, BC = 9 cm and $A\hat{B}C = 120°$.

(a) Find AC.

(b) Find $B\hat{A}C$.

5. The following diagram shows a triangle ABC, where $A\hat{C}B$ is 90°, AB = 3, AC = 2 and $B\hat{A}C$ is θ.



(a) Show that $\sin \theta = \frac{\sqrt{5}}{3}$.

(b) Show that $\sin 2\theta = \frac{4\sqrt{5}}{9}$.

(c) Find the **exact** value of $\cos 2\theta$.

6. (a) Show that $4 - \cos 2\theta + 5 \sin \theta = 2 \sin^2 \theta + 5 \sin \theta + 3$.

(b) **Hence**, solve the equation $4 - \cos 2\theta + 5 \sin \theta = 0$ for $0 \leq \theta \leq 2\pi$.

7. The diagram below shows triangle PQR. The length of [PQ] is 7 cm, the length of [PR] is 10 cm, and $\hat{PQR}$ is 75°.

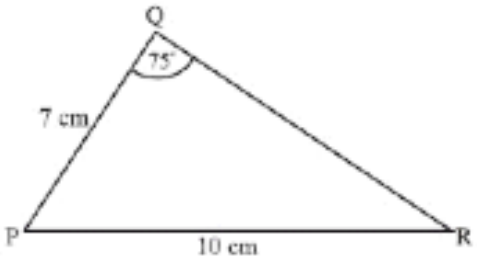


diagram not to scale

(a) Find $\hat{PQR}$.

(b) Find the area of triangle PQR.

8. The straight line with equation $y = \frac{3}{4}x$ makes an acute angle θ with the x-axis.

(a) Write down the value of $\tan \theta$.

(b) Find the value of

(i) $\sin 2\theta$;

(ii) $\cos 2\theta$.

9. The diagram below shows a quadrilateral ABCD with obtuse angles $\hat{ABC}$ and $\hat{ADC}$.

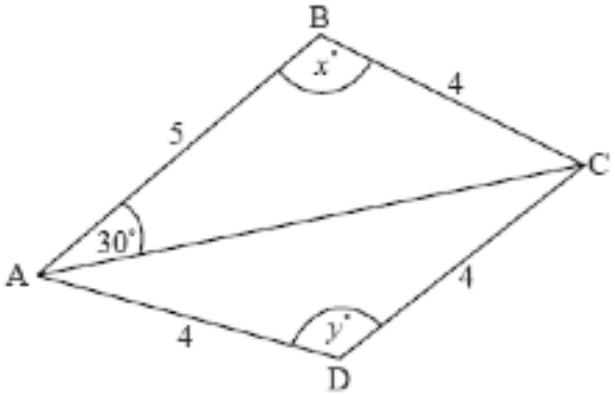


diagram not to scale

AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm, $\hat{BAC} = 30°$, $\hat{ABC} = x°$, $\hat{ADC} = y°$.

(a) Use the cosine rule to show that $AC = \sqrt{41 - 40\cos x}$.

(b) Use the sine rule in triangle ABC to find another expression for AC.

(c) (i) Hence, find x, giving your answer to two decimal places.

(ii) Find AC.

(d) (i) Find y.

(ii) Hence, or otherwise, find the area of triangle ACD.