SATPREP
Assignment : Exponent and Logarithm

1. Find the **exact** value of x satisfying the equation

    $$(3^x)(4^{2x+1}) = 6^{x+2}.$$

    Give your answer in the form $\dfrac{\ln a}{\ln b}$ where $a, b \in \mathbb{Z}$.

2. The solution of $2^{2x+3} = 2^{x+1} + 3$ can be expressed in the form $a + \log_2 b$ where $a, b \in \mathbb{Z}$. Find the value of a and of b.

3. Find $\displaystyle\sum_{n=1}^{15} a_n^2$ where $a_n = \ln x^n$.

4. (a) Solve the equation $2(4^x) + 4^{-x} = 3$.

    (b) (i) Solve the equation $a^x = e^{2x+1}$ where $a > 0$, giving your answer for x in terms of a.

    (ii) For what value of a does the equation have no solution?

5. Solve the equation $9 \log_5 x = 25 \log_x 5$, expressing your answers in the form $5^{\frac{p}{q}}$, where $p, q \in \mathbb{Z}$.

6. Solve, for x, the equation $\log_2 (5x^2 - x - 2) = 2 + 2 \log_2 x$.

7. Solve $2 (\ln x)^2 = 3\ln x - 1$ for x. Give your answers in **exact** form.

8. Solve $2(5^{x+1}) = 1 + \dfrac{3}{5^x}$, giving the answer in the form $a + \log_5 b$, where $a, b \in \mathbb{Z}$.

9. The function f is defined for $x > 2$ by $f(x) = \ln x + \ln (x - 2) - \ln (x^2 - 4)$.

    (a) Express $f(x)$ in the form $\ln\left(\dfrac{x}{x + a}\right)$.

    (b) Find an expression for $f^{-1}(x)$.

10. Solve $\log_{16} \sqrt[3]{100 - x^2} = \dfrac{1}{2}$.