SATPREP

Assignment : Sequence and Series (Difficult)

1. The second term of an arithmetic sequence is 7. The sum of the first four terms of the arithmetic sequence is 12. Find the first term, a, and the common difference, d, of the sequence.

2. The ratio of the fifth term to the twelfth term of a sequence in an arithmetic progression is $\frac{6}{13}$. If each term of this sequence is positive, and the product of the first term and the third term is 32, find the sum of the first 100 terms of this sequence.

3. The sum of the first n terms of a series is given by

$$S_n = 2n^2 - n, \text{ where } n \in \mathbb{Z}^+.$$

(a) Find the first three terms of the series.

(b) Find an expression for the n^{th} term of the series, giving your answer in terms of n.

4. Let S_n be the sum of the first n terms of an arithmetic sequence, whose first three terms are u_1, u_2 and u_3. It is known that $S_1 = 7$, and $S_2 = 18$.

(a) Write down u_1.

(b) Calculate the common difference of the sequence.

(c) Calculate u_4.

5. The nth term, u_n, of a geometric sequence is given by $u_n = 3(4)^{n+1}$, $n \in \mathbb{Z}^+$.

(a) Find the common ratio r.

(b) Hence, or otherwise, find S_n, the sum of the first n terms of this sequence.

6. A geometric sequence has all positive terms. The sum of the first two terms is 15 and the sum to infinity is 27. Find the value of

(a) the common ratio;

(b) the first term.

7. The sum of the first n terms of an arithmetic sequence $\{u_n\}$ is given by the formula $S_n = 4n^2 - 2n$. Three terms of this sequence, u_2, u_m and u_{32}, are consecutive terms in a geometric sequence. Find m.

8. The three terms a, 1, b are in arithmetic progression. The three terms 1, a, b are in geometric progression. Find the value of a and of b given that $a \neq b$.

9. The first four terms of an arithmetic sequence are 2, $a - b$, $2a + b + 7$, and a – 3b, where a and b are constants. Find a and b.

10. Arturo goes swimming every week. He swims 200 metres in the first week. Each week he swims 30 metres more than the previous week. He continues for one year (52 weeks).

(a) How far does Arturo swim in the final week?

(b) How far does he swim altogether?

11. Each day a runner trains for a 10 km race. On the first day she runs 1000 m, and then increases the distance by 250 m on each subsequent day.

(a) On which day does she run a distance of 10 km in training?

(b) What is the total distance she will have run in training by the end of that day? Give your answer exactly.

12. $1000 is invested at the beginning of each year for 10 years.

The rate of interest is fixed at 7.5% per annum. Interest is compounded annually.

Calculate, giving your answers to the nearest dollar

(a) how much the first $1000 is worth at the end of the ten years;

(b) the total value of the investments at the end of the ten years.