Problem 0580/41/M/J/20/ Q10B

(b) A curve has the equation $y = x^3 + 8x^2 + 5x$.

(i) Work out the coordinates of the two turning points.

$$\frac{dy}{dx} = 3x^2 + 16x + 5$$

$$\frac{dy}{dx} = 0 \quad \therefore \quad 3x^2 + 16x + 5 = 0$$

$$x = -\frac{1}{3} \qquad x = -5$$

$$y = \left(-\frac{1}{3}\right)^3 + 8\left(-\frac{1}{3}\right)^2 + 5\left(-\frac{1}{3}\right) = -\frac{22}{27} \qquad \left(-\frac{1}{3}, -\frac{22}{27}\right)$$

$$y = (-5)^3 + 8(-5)^2 + 5(-5) = 50 \qquad (-5, 50)$$

(ii) Determine whether each of the turning points is a maximum or a minimum.
Give reasons for your answers.

$$\frac{d^2y}{dx^2} = 6x + 16$$

$$x = -\frac{1}{3} \qquad \frac{d^2y}{dx^2} = 6\left(-\frac{1}{3}\right) + 16 = 14 > 0$$

hence turning point $\left(-\frac{1}{3}, -\frac{22}{27}\right)$
will be minimum

$$x = -5 \qquad \frac{d^2y}{dx^2} = 6 \times -5 + 16 = -14 < 0$$

hence turning point $(-5, 50)$
will be maximum.