Problem : 0580/42/F/M/23/Q12C

A curve has equation $y = x^3 + ax + b$.
The stationary points of the curve have coordinates $(2, k)$ and $(-2, 10 - k)$.

Work out the value of a, the value of b and the value of k.

Sol

$$\frac{dy}{dx} = 3x^2 + a$$

$$0 = 3(2)^2 + a$$

$$a = -12$$

$$y = x^3 - 12x + b$$

$(2, k)$

$$k = 2^3 - 12 \times 2 + b$$

$$k = 8 - 24 + b$$

$$k - b = -16 \quad\text{---(i)}$$

$-2, \ 10 - k$

$$10 - k = (-2)^3 - 12 \times -2 + b$$

$$10 - k = -8 + 24 + b$$

$$-k = -10 - 8 + 24 + b$$

$$-b - k = 6 \quad\text{---(ii)}$$

by solving eq (i) & (ii)

So $\quad b = 5 \quad k = -11$