

Problem : 0580/42/M/J/23/Q12

A curve has equation $y = x^3 - kx^2 + 1$.

When $x = 2$, the gradient of the curve is 6.

(a) Show that $k = 1.5$.

$$\begin{aligned}\frac{dy}{dx} &= 3x^2 - k \times 2x \\ &= 3x^2 - 2kx \\ 6 &= 3(2)^2 - 2k(2) \\ 6 &= 12 - 4k \\ -6 &= -4k \\ k &= \frac{6}{4} = 1.5\end{aligned}$$

(b) Find the coordinates of the two stationary points of $y = x^3 - 1.5x^2 + 1$.
You must show all your working.

$$\begin{aligned}y &= x^3 - 1.5x^2 + 1 \\ \frac{dy}{dx} &= 3x^2 - 1.5 \times 2x \\ 0 &= 3x^2 - 3x \\ 0 &= 3x(x-1) \\ x = 0 & \quad x = 1 \\ y = 1 & \quad y = 1 - 1.5 + 1 \\ & \quad = 0.5 \\ (0, 1) & \quad (1, 0.5)\end{aligned}$$