Problem : 0580/43/M/J/23/Q 12

The equation of a curve is $y = x^4 - 8x^2 + 5$.

(a) Find the derivative, $\left(\frac{dy}{dx}\right)$, of $y = x^4 - 8x^2 + 5$.

$$\frac{dy}{dx} = 4x^3 - 8 \times 2x + 0$$
$$= 4x^3 - 16x$$

(b) Find the coordinates of the three turning points.
You must show all your working.

$$\frac{dy}{dx} = 0 \quad 4x^3 - 16x = 0$$
$$4x(x^2 - 4) = 0$$
$$x = 0 \quad x^2 - 4 = 0$$
$$x = 0 \quad x^2 = 4$$
$$x = 2 \quad x = -2$$

$$x = 0 \quad y = x^4 - 8x^2 + 5 \quad y = 0^4 - 8(0) + 5$$
$$= 5$$
$$x = 2 \quad y = 2^4 - 8(2)^2 + 5 = -11$$
$$x = -2 \quad y = (-2)^4 - 8(-2)^2 + 5 = -11$$
$$(0, 5) \quad (2, -11) \quad (-2, -11)$$

(c) Determine which one of these turning points is a maximum.
Justify your answer.

$$\frac{d^2y}{dx^2} = 4 \times 3 \times x^2 - 16$$
$$= 12x^2 - 16$$

$$x = 0 \quad \frac{d^2y}{dx^2} = 12(0)^2 - 16 = -16 < 0$$
~~Maximum~~ at $(0, 5)$
maximum

$$x = 2 \quad \frac{d^2y}{dx^2} = 12(2)^2 - 16 = 32 > 0$$
minimum

$$x = -2 \quad \frac{d^2y}{dx^2} = 12(-2)^2 - 16 = 32 > 0$$
minimum