

Problem : 0580/43/O/N/22/ Q9B

- (b) (i) Find the derivative, $\frac{dy}{dx}$, of $y = 5 + 8x - \frac{4}{3}x^3$.

$$\begin{aligned}\frac{dy}{dx} &= 0 + 8 - \frac{4}{3} \times 3x^2 \\ &= 8 - 4x^2\end{aligned}$$

- (ii) Find the gradient of $y = 5 + 8x - \frac{4}{3}x^3$ at $x = -1$.

$$\begin{aligned}\frac{dy}{dx} &= 8 - 4(-1)^2 \\ &= 8 - 4 = 4\end{aligned}$$

- (iii) A tangent is drawn to the graph of $y = 5 + 8x - \frac{4}{3}x^3$.

The gradient of the tangent is -28.

Find the coordinates of the two possible points where this tangent meets the graph.

$$\begin{aligned}\frac{dy}{dx} &= 8 - 4x^2 \\ -28 &= 8 - 4x^2 \\ +36 &= +4x^2 \\ x^2 &= 9\end{aligned}$$

$$x = 3 \quad x = -3$$

$$y = 5 + 8x - \frac{4}{3}x^3$$

$$\begin{aligned}y &= 5 + 8 \times 3 - \frac{4}{3} \times \frac{27}{3} \\ &= -7\end{aligned}$$

$$(3, -7)$$

$$y = 5 + 8x - 3 - \frac{4}{3}(-3)^3 = 17$$

$$(-3, 17)$$