Subject – Math AA(Higher Level)
Topic - Statistics and Probability
Year - May 2021 – Nov 2024
Paper -3
Questions

Question 1

(a) 6^3 OR $6\times6\times6$ ***A1***

Note: Accept a labelled diagram that clearly illustrates correct application of the multiplication principle leading to 216.

$=216$ ***AG***

[1 mark]

(b) **EITHER**

attempts to find Δ ***(M1)***

$\Delta\left(=\left(4^2-4(1)(4)\right)\right)=0$ ***A1***

OR

attempts to solve $x^2+4x+4=0$ ***(M1)***

$\left((x+2)^2=0\Rightarrow\right)x=-2$ ***A1***

OR

attempts to express $x^2+4x+4(=0)$ as a perfect square ***(M1)***

$(x+2)^2(=0)$ is a perfect square ***A1***

OR

a graph of $y=x^2+4x+4$ with the vertex touching the x-axis at $x=-2$ ***A2***

THEN

graph of f has only one x-intercept ***AG***

[2 marks]

Note: In parts (c) – (f), $(a,b,c)=(1,2,1)$, for example, represents an ordered 3-tuple $a=1, b=2$ and $c=1$.

(c) recognizes that $b^2-4ac=0$ (or equivalent) ***(M1)***

EITHER

attempts to use $\frac{b^2}{ac}=4$ $\left(\frac{b^2}{4}=ac\right)$ ***(M1)***

determines one value of b from $b=2,4$ or 6 only (seen anywhere) OR one value of ac from $ac=1,4$ or 9 only (seen anywhere) ***(A1)***

OR

attempts to find a possible value of b ***(M1)***

determines one value of b from $b=2,4$ or 6 only (seen anywhere) ***(A1)***

OR

recognizes that b^2 must be a multiple of 4 OR b must be a multiple of 2 ***(M1)***

determines one value of b from $b=2,4$ or 6 only (seen anywhere) ***(A1)***

OR

attempts to find a possible value of ac ***(M1)***

determines one value of ac from $ac=1,4$ or 9 only (seen anywhere) ***(A1)***

THEN

$b=2$ and $ac=1$:

$(a,b,c)=(1,2,1)$ OR 1 possible way OR $\frac{1}{216}$ ***A1***

$b=4$ and $ac=4$:

$(a,b,c)=(1,4,4),(4,4,1),(2,4,2)$ OR 3 possible ways OR $\frac{3}{216}$ ***A1***

$b=6$ and $ac=9$:

$(a,b,c)=(3,6,3)$ OR 1 possible way OR $\frac{1}{216}$ ***A1***

therefore the required probability is $\frac{1}{216}+\frac{3}{216}+\frac{1}{216}$

$=\frac{5}{216}$ ***AG***

[6 marks]

(d) recognizes that $b^2 - 4ac > 0$ (or equivalent eg. $\frac{b^2}{4} > ac$) ***(M1)***

maximum value of b^2 is 36 OR maximum value of ac is 8 ***(A1)***

Note: The above ***(A1)*** is independent of the ***(M1)***.

$ac = 1, 2, 3, 4, 5, 6, 8$ ***A1***

[3 marks]

(e) (i) $ac = 1$ $\left(b^2 > 4\right)$

$b = 3, 4, 5, 6$ OR 1×4 (quadratics) OR $6 - 2$ (quadratics) ***A1***

there are four quadratic functions ***AG***

[1 mark]

(ii)

$ac = 2$ $\left(b^2 > 8\right)$

$b = 3, 4, 5, 6$ ***(A1)***

Note: Award ***(A1)*** for referencing their result shown in part (e) (i).

EITHER

$(a, b, c) = (1, 3, 2), (1, 4, 2), (1, 5, 2), (1, 6, 2), (2, 3, 1), (2, 4, 1), (2, 5, 1), (2, 6, 1)$ ***A1***

Note: Award ***A1*** for listing the eight quadratic expressions.

OR

2×4 (quadratics) ***A1***

THEN

there are eight quadratics functions ***AG***

[2 marks]

(f) **METHOD 1**

varies ac $(ac \neq 1,2)$ and determines possible values of b such that $\Delta > 0$ ***(M1)***

correctly determines one of the following five cases ***(A1)***

correctly determines a further two of the following five cases ***(A1)***

correctly determines the remaining two cases ***(A1)***

case 1: $ac = 3$ $(b^2 > 12 \Rightarrow b = 4,5,6)$

$(a,b,c) = (1,4,3),(1,5,3),(1,6,3),(3,4,1),(3,5,1),(3,6,1)$ OR

6 possible ways OR $\frac{6}{216}$

case 2: $ac = 4$ $(b^2 > 16 \Rightarrow b = 5,6)$

$(a,b,c) = (1,5,4),(1,6,4),(2,5,2),(2,6,2),(4,5,1),(4,6,1)$ OR

6 possible ways OR $\frac{6}{216}$

case 3: $ac = 5$ $(b^2 > 20 \Rightarrow b = 5,6)$

$(a,b,c) = (1,5,5),(1,6,5),(5,5,1),(5,6,1)$ OR 4 possible ways OR $\frac{4}{216}$

case 4: $ac = 6$ $(b^2 > 24 \Rightarrow b = 5,6)$

$(a,b,c) = (1,5,6),(2,5,3),(3,5,2),(6,5,1),(1,6,6),(2,6,3),(3,6,2),(6,6,1)$

OR 8 possible ways OR $\frac{8}{216}$

case 5: $ac = 8$ $(b^2 > 32 \Rightarrow b = 6)$

$(a,b,c) = (2,6,4),(4,6,2)$ OR 2 possible ways OR $\frac{2}{216}$

adds their probabilities ***(M1)***

Note: Award ***(M1)*** for adding at least 3 of their probabilities (denominator 216).

$$(p =)\ \frac{4}{216}+\frac{8}{216}+\frac{6}{216}+\frac{6}{216}+\frac{4}{216}+\frac{8}{216}+\frac{2}{216}$$

$$(= 0.0185... + 0.0370... + 0.0277... + 0.0277... + 0.0185... + 0.0370... + 0.0092...)$$

$$= \frac{38}{216}\ \left(= \frac{19}{108}, = 0.176\right)$$ ***A1***

METHOD 2

varies $b^2(\neq 1,4)$ OR $b(\neq 1,2)$ and determines possible values of ac such that $\Delta > 0$ ***(M1)***

correctly determines one of the following four cases ***(A1)***

correctly determines another case from the following four cases ***(A1)***

correctly determines the remaining two cases ***(A1)***

case 1: $b^2 = 9$ $(b=3)$ $(ac = 1,2)$

$(a,b,c) = (1,3,1),(1,3,2),(2,3,1)$ OR 3 possible ways OR $\frac{3}{216}$

case 2: $b^2 = 16$ $(b=4)$ $(ac = 1,2,3)$

$(a,b,c) = (1,4,1),(1,4,2),(2,4,1),(1,4,3),(3,4,1)$ OR 5 possible ways OR $\frac{5}{216}$

case 3: $b^2 = 25$ $(b=5)$ $(ac = 1,2,3,4,5,6)$

$$(a,b,c) = (1,5,1),(1,5,2),(2,5,1),(1,5,3),(3,5,1),(1,5,4),(2,5,2)$$
$$(4,5,1),(1,5,5),(5,5,1),(1,5,6),(2,5,3),(3,5,2),(6,5,1)$$

OR 14 possible ways OR $\frac{14}{216}$

case 4: $b^2 = 36$ $(b=6)$ $(ac = 1,2,3,4,5,6,8)$

$$(a,b,c) = (1,6,1),(1,6,2),(2,6,1),(1,6,3),(3,6,1),(1,6,4),(2,6,2),(4,6,1)$$
$$(1,6,5),(5,6,1),(1,6,6),(2,6,3),(3,6,2),(6,6,1),(2,6,4),(4,6,2)$$

OR 16 possible ways OR $\frac{16}{216}$

adds their probabilities ***(M1)***

Note: Award ***(M1)*** for adding at least 3 of their probabilities (denominator 216).

$$(p=)\ \frac{3}{216}+\frac{5}{216}+\frac{14}{216}+\frac{16}{216}$$

$$(=0.013889\ldots+0.023148\ldots+0.064815\ldots+0.074074\ldots)$$

$$=\frac{38}{216}\ \left(=\frac{19}{108}, =0.176\right)$$ ***A1***

METHOD 3

varies b^2 OR b and determines possible values of ac such that $\Delta < 0$ ***(M1)***

correctly determines two of the following six cases ***(A1)***

correctly determines a further two of the following six cases ***(A1)***

correctly determines the remaining two cases ***(A1)***

case 1: $b^2 = 1$ $(b = 1)$ 36 possible ways OR $\frac{36}{216}$

case 2: $b^2 = 4$ $(b = 2)$ 35 possible ways OR $\frac{35}{216}$

case 3: $b^2 = 9$ $(b = 3)$ 33 possible ways OR $\frac{33}{216}$

case 4: $b^2 = 16$ $(b = 4)$ 28 possible ways OR $\frac{28}{216}$

case 5: $b^2 = 25$ $(b = 5)$ 22 possible ways OR $\frac{22}{216}$

case 6: $b^2 = 36$ $(b = 6)$ 19 possible ways OR $\frac{19}{216}$

$(p =)\ 1 - \left(\frac{36}{216} + \frac{35}{216} + \frac{33}{216} + \frac{28}{216} + \frac{22}{216} + \frac{19}{216} + \frac{5}{216}\right)$ ***(M1)***

$$\left(\begin{array}{l} = 1 - \\ (0.16666... + 0.16203... + 0.15277... + 0.12962... + 0.10185... + 0.087962... + 0.023148...) \end{array} \right)$$

Note: Award ***(M1)*** for adding at least 3 of their probabilities inside the above bracket (denominator 216).

$= \frac{38}{216}$ $\left(= \frac{19}{108}, = 0.176\right)$ ***A1***

[6 marks]

Question 2

. (a) (i) $0.9\times0.9\times0.1$ ***(M1)***

Note: Award ***M1*** for 0.9 seen as part of a multiplication.

$=0.081$ ***A1***

[2 marks]

(ii) let $A=$ number of boosts in the first 6 actions

METHOD 1

Recognition of binomial distribution ***(M1)***

with parameters $n=6$ and $p=0.1$ ***(A1)***

$(P(A\geq1)=)0.469 \quad (0.468558...)$ ***A1***

Note: Award ***(M1)(A1)A0*** for an answer of 0.114 (from finding $P(A>1)$).

METHOD 2

$(\mathrm{P}(A\geq1)=)\ 1-\mathrm{P}(\text{no boost in first } 6)$ ***(M1)***

$=1-0.9^6$ ***(A1)***

$=0.469\ (0.468558...)$ ***A1***

METHOD 3

$0.1+0.9\times0.1+0.9^2\times0.1+0.9^3\times0.1+0.9^4\times0.1+0.9^5\times0.1$ ***(M1)(A1)***

Note: Award ***(M1)*** for at least 3 correct terms in a sum.

$=0.469\ (0.468558...)$ ***A1***

[3 marks]

(g) recognizes that $4Z^2-4>0$ $(Z^2>1)$ ***(M1)***

probability of two x-intercepts is

EITHER

$\text{P}(|Z|>1)$ ***(A1)***

OR

$\text{P}(Z<-1)$ or $\text{P}(Z>1)$ (can be shown on a labelled diagram) ***(A1)***

$=0.158655...+0.158655...$

OR

$1-\text{P}(-1\le Z\le 1)$ (can be shown on a labelled diagram) ***(A1)***

$=1-0.682689...$

THEN

$=0.317310...$

$=0.317$ ***A1***

[3 marks]

(h) attempts to solve $X_1>0.5$ for Z ***(M1)***

$-1.25<Z\le -1$ ***(A1)(A1)***

> **Note:** Award ***(M1)(A1)*** for obtaining $Z=-1.25$ from solving $X_1=0.5$ and award ***(A1)*** for stating the correct inequality.
> Award ***(M1)(A1)(A1)*** for $-1.25<Z<-1$.
> Award ***(M1)(A1)(A0)*** for $-1.25<Z$.
> Award ***(M1)*** for rearranging to form $-\sqrt{Z^2-1}=Z+0.5$ and then attempting to square both sides $Z^2-1=(Z+0.5)^2$ $(=Z^2+Z+0.25)$.

attempts to calculate their $\text{P}(X_1, X_2 \text{ both } >0.5)$ ***(M1)***

$\text{P}(-1.25<Z\le -1)=0.053005...$ ***(A1)***

attempts to calculate their $\text{P}(X_1, X_2 \text{ both } >0.5\,|\,x\text{-intercepts})$ ***(M1)***

$=\dfrac{0.053005...}{0.317310...}$

$=0.167$ ***A1***

[7 marks]

Total [31 marks]

Question 2

. (a) (i) $0.9\times0.9\times0.1$ ***(M1)***

Note: Award ***M1*** for 0.9 seen as part of a multiplication.

$=0.081$ ***A1***

[2 marks]

(ii) let $A=$ number of boosts in the first 6 actions

METHOD 1

Recognition of binomial distribution ***(M1)***

with parameters $n=6$ and $p=0.1$ ***(A1)***

$(P(A\geq1)=)0.469\ \ (0.468558...)$ ***A1***

Note: Award ***(M1)(A1)A0*** for an answer of 0.114 (from finding $P(A>1)$).

METHOD 2

$(\text{P}(A\geq1)=)\ 1-\text{P}(\text{no boost in first } 6)$ ***(M1)***

$=1-0.9^6$ ***(A1)***

$=0.469\ (0.468558...)$ ***A1***

METHOD 3

$0.1+0.9\times0.1+0.9^2\times0.1+0.9^3\times0.1+0.9^4\times0.1+0.9^5\times0.1$ ***(M1)(A1)***

Note: Award ***(M1)*** for at least 3 correct terms in a sum.

$=0.469\ (0.468558...)$ ***A1***

[3 marks]

(b) (i) **EITHER**

there will be $x-1$ actions not boosted, followed by one boosted action ***A1***

OR

The x^{th} action is boosted and all previous actions are not boosted **A1**

OR

$\underbrace{(1-p)\times(1-p)\times\ldots\times(1-p)}_{(x-1)\text{ times}}\times p$ ***A1***

Note: For the **OR** method, the order of the operations must be clear (accept also a tree diagram etc.).

THEN

$p(1-p)^{x-1}$ ***AG***

[1 mark]

(ii) $\mathrm{E}(X)=\sum_{x=1}^{\infty} xp(1-p)^{x-1}$ ***A1***

[1 mark]

(c) (i) LHS:

$a+2ar+3ar^2+\dots$ **A1**

Note: Award A1 for writing LHS $=\sum ar^n$ and differentiating to get $=\sum anr^{n-1}$.

RHS:

attempt to differentiate $a(1-r)^{-1}$ or use quotient rule ***(M1)***

$\frac{a}{(1-r)^2}$ **A1**

$\sum_{n=1}^{\infty} nar^{n-1}=\frac{a}{(1-r)^2}$ **A1**

Note: To award the final ***A1*** "$\sum_{n=1}^{\infty} nar^{n-1}$" must be seen either as part of the LHS working or as part of the final answer and all previous marks must be awarded.

[4 marks]

(ii) recognition that $a=p$ and $r=1-p$ (and $x=n$) ***(M1)***

$\mathrm{E}(X)=\frac{p}{(1-(1-p))^2}$ **A1**

Note: Award ***A0 FT*** if their answer does not lead to the ***AG***.

$=\frac{1}{p}$ ***AG***

[2 marks]

(d) $\mathrm{E}(X)=10$, $\mathrm{Var}(X)=90$ ***A1A1***

[2 marks]

(e) $0.8\times0.6\times0.6$ ***(M1)A1***

Note: Award ***M1*** for evidence of P("not boosted") changing, e.g. a labelled tree diagram and/or 0.8 AND 0.6 seen.

$= 0.288$ ***AG***

[2 marks]

(f) after four actions that are not boosted, the probability that the next action is boosted is 1 (and $p=1$ is a certainty) ***R1***

Note: Accept "when $Y=5$, the probability of a boost is 1".
Do not accept "on the 5^{th} action the probability of a boost is 1", unless there is reference to the previous actions being not boosted.

[1 mark]

(g) (i) $m=0.32,\ n=0.1536\ \ (0.154)$ ***A1A1***

[2 marks]

(ii) attempt to multiply outcomes by probabilities and find the sum ***(M1)***

$0.2+2\times0.32+3\times0.288+4\times0.1536+5\times0.0384$ ***A1***

Note: The ***A1*** can only be awarded if m and n are correct and the exact value of n is used.

$= 2.5104$ ***AG***

[2 marks]

(iii) $0.2+2^2m+3^2\times0.288+4^2\times n+5^2\times0.0384-2.5104^2$ ***(A1)***

$= 1.19\ (1.18749\ldots)$ ***A1***

Note: Award ***A1A1*** for a correct answer found using the GDC.

[2 marks]

(h) (i) $\left(p = \frac{1}{2.5104}\right) = 0.398\ (0.398342...)$ ***A1***

[1 mark]

(ii) $\text{Var}(X) = 3.79\ (3.79170...)$ ***A1***

Note: Award ***A1*** for an answer of 3.80 (3.80040…) (from using their 3sf answer from part (i).) Condone 3.8.

[1 mark]

(iii) since $\text{Var}(X) > \text{Var}(Y)$, 2^{nd} model provides more consistent experience ***R1***

Note: Only award ***R1 FT*** if both their variances are positive.

[1 mark]

Total [27 marks]