

Subject - Math AI(Higher Level)
Topic - Functions
Year - May 2021 – Nov 2024
Paper -3
Answers

Question 1

(a) (i) $Q(t) = 3090t - 54000$ (3094.27... t - 54042.3...)

A1A1

Note: Award at most **A1A0** if answer is not an equation. Award **A1A0** for an answer including either x or y .

[2 marks]

(ii) 0.755 (0.754741...)

A1

[1 mark]

- (iii) t is not a random variable **OR** it is not a (bivariate) normal distribution
OR data is not a sample from a population
OR data appears nonlinear
OR r only measures linear correlation

R1

Note: Do not accept " r is not large enough".

[1 mark]

- (b) (i) attempt to separate variables

(M1)

$$\int \frac{1}{Q} dQ = \int \beta N dt$$

$$\ln|Q| = \beta Nt + c$$

A1A1A1

Note: Award **A1** for LHS, **A1** for βNt , and **A1** for $+c$.

Award full marks for $Q = e^{\beta Nt+c}$ **OR** $Q = Ae^{\beta Nt}$.

Award **M1A1A1A0** for $Q = e^{\beta Nt}$

[4 marks]

- (ii) attempt at exponential regression

(M1)

$$Q = 1.15e^{0.292t} \quad (Q = 1.14864...e^{0.292055...t})$$

A1

OR

- attempt at exponential regression

(M1)

$$Q = 1.15 \times 1.34^t \quad (1.14864... \times 1.33917...^t)$$

A1

Note: Condone answers involving y or x . Condone absence of " $Q =$ "

Award **M1A0** for an incorrect answer in correct format.

[2 marks]

(iii) 0.999 (0.999431...)

A1

[1 mark]

(iv) comparing something to do with R^2 and something to do with r

M1

Note: Examples of where the **M1** should be awarded:

$$R^2 > r$$

$$R > r$$

$$0.999 > 0.755$$

$$0.999 > 0.755^2 \quad (= 0.563)$$

The "correlation coefficient" in the exponential model is larger.

Model B has a larger R^2

Examples of where the **M1** should **not** be awarded:

The exponential model shows better correlation (since not clear how it is being measured)

Model 2 has a better fit

Model 2 is more correlated

an unambiguous comparison between R^2 and r^2 or R and r leading to the conclusion that the model in part (b) is more suitable / better

A1

Note: Condone candidates claiming that R is the "correlation coefficient" for the non-linear model.

[2 marks]

(v) it suggests that there will be more infected computers than the entire population

R1

Note: Accept any response that recognizes unlimited growth.

[1 mark]

(c) $1.15e^{0.292t} = 2.3$ **OR** $1.15 \times 1.34^t = 2.3$ **OR** $t = \frac{\ln 2}{0.292}$ **OR** using the model to find two specific times with values of $Q(t)$ which double
 $t = 2.37$ (days)

M1

A1

Note: Do not **FT** from a model which is not exponential. Award **MOA0** for an answer of 2.13 which comes from using (10, 20) from the data or any other answer which finds a doubling time from figures given in the table.

[2 marks]

(d) an attempt to calculate β for city X

(M1)

$$\beta = \frac{0.292055...}{2.6 \times 10^6} \quad \text{OR} \quad \beta = \frac{\ln 1.33917...}{2.6 \times 10^6}$$

$$= 1.12328... \times 10^{-7}$$

this is larger than 9.64×10^{-8} so the virus spreads more easily in city X

A1

R1

Note: It is possible to award **M1A0R1**.

Condone "so the virus spreads faster in city X" for the final **R1**.

[3 marks]

(e) $a = 38.3, b = 3086.1$

A1A1

Note: Award **A1A0** if values are correct but not to 1 dp.

[2 marks]

(f) (i) $\frac{Q'}{Q} = 0.42228 - 2.5561 \times 10^{-6} Q$

(A1)(A1)

Note: Award **A1** for each coefficient seen – not necessarily in the equation. Do not penalize seeing in the context of y and x .

identifying that the constant is k **OR** that the gradient is $-\frac{k}{L}$

(M1)

therefore $k = 0.422$ (0.422228...)

A1

$$\frac{k}{L} = 2.5561 \times 10^{-6}$$

$$L = 165000 \text{ (165205)}$$

A1

Note: Accept a value of L of 164843 from use of 3 sf value of k , or any other value from plausible pre-rounding. Allow follow-through **within** the question part, from the equation of their line to the final two **A1** marks.

[5 marks]

(ii) recognizing that their L is the eventual number of infected

(M1)

$$\frac{165205...}{2600000} = 6.35\% \text{ (6.35403...%)}$$

A1

Note: Accept any final answer consistent with their answer to part (f)(i) unless their L is less than 120146 in which case award at most **M1A0**.

[2 marks]

[Total 28 marks]

Question 2

(a) $C = kd$ (M1)

$0.80 = 0.5k$ OR $\frac{C}{d} = \frac{0.8}{0.5}$ (A1)

$k = 1.6$

$C = 1.6d$ OR $C = \frac{d}{0.625}$ A1

Note: For the final A1 do not accept $C = \frac{0.8}{0.5}d$ or a correct equation which does not have C as the subject.

[3 marks]

(b) $d = \frac{0.96}{1.6}$ M1

Note: Award M1 for the substitution of $C = 0.96$ into a correct equation, award M0 for substitution of $d = 0.6$.

$= 0.6$

AG

[1 mark]

(c) attempt at using Pythagoras

$d^2 = h^2 + (2r)^2$

$r^2 = \frac{1}{4}(d^2 - h^2)$

M1

(A1)

A1

[3 marks]

(d) $V = \pi r^2(2h)$ (A1)

$V = \frac{\pi}{4}(d^2 - h^2)(2h)$ M1

Note: The M1 is for the substitution of their expression for r^2 into their formula for V .

$V = \frac{\pi}{2}(d^2h - h^3)$

AG

[2 marks]

$$(e) \quad (i) \quad V = \frac{\pi}{2}(d^2h - h^3)$$

$$= \frac{\pi}{2}(0.6^2 \times 0.4 - 0.4^3)$$

(M1)

Note: The **M1** is for substituting correct values of both d and h in the formula from part (d).

$$= 0.126 \text{ (m}^3\text{)} \left(0.12566\dots, \frac{\pi}{25}, 0.04\pi \right)$$

A1

[2 marks]

$$(ii) \quad V = \frac{\pi}{2}(0.36h - h^3)$$

M1

Note: Award **M1** for substitution of $d = 0.6$. This may be seen anywhere.

$$\frac{dV}{dh} = \frac{\pi}{2}(0.36 - 3h^2) = 0$$

A1

Note: Award **A1** for correct differentiation.

$$h^2 = \frac{0.36}{3} (= 0.12)$$

A1

Note: Do not award the final **A1** if the working is done with approximate values or if $\sqrt{0.12}$ is substituted into $\frac{dV}{dh}$.

$$h = \sqrt{0.12}$$

AG

[3 marks]

(iii) substituting $h = \sqrt{0.12}$ into equation for V OR use of graph

(M1)

$$V = \frac{\pi}{2}(0.36 - 0.12)\sqrt{0.12}$$

$$0.131 \text{ (m}^3\text{)} \quad (0.130593\dots, 0.0416\pi)$$

A1

[2 marks]

- (f) x -coordinate of S is 0.4 (A1)
 let the y -coordinate be y_s
 attempt at Pythagoras (M1)
 $(y_s + 0.2)^2 = 0.6^2 - 0.4^2$
 $y_s + 0.2 = \sqrt{0.2}$
 $y_s = 0.247$ (0.247213...) (A1)

Note: The (M1) mark can be implied by a y -coordinate of 0.447 or 0.647 seen.

any valid method to find equation (M1)
 e.g. quadratic regression, vertex form, simultaneous equations

Note: Award only if the student has found three points on the curve.

EITHER

$y = -0.295x^2 + 0.236x + 0.2$ (A2)
 $(y = -0.295081\dots x^2 + 0.236065\dots x + 0.2)$

Note: Award A1 if one coefficient ($-0.295081\dots$ or $0.236065\dots$) is correct or if " $y =$ " is missing, A2 for completely correct equation.
 Award A1 for $y = -0.294x^2 + 0.235x + 0.2$ obtained from using the 3 sf value of y_s .

OR

$y = -0.295(x - 0.4)^2 + 0.247$ (A2)
 $(y = -0.295081\dots(x - 0.4)^2 + 0.247213\dots)$

Note: Award A1 for -0.295 , A2 for completely correct equation.

[6 marks]

(g) volume = $\pi \int_0^{0.8} (-0.295081\dots x^2 + 0.236065\dots x + 0.2)^2 dx$ (M1)

Note: Award M1 for the minimum of an integral with the correct limits and their function squared.

= 0.135 (0.135161...) (A1)

0.135 > 0.131 (R1)

Note: Award R1 independently of the previous marks for a correct comparison of their (clearly stated) volume with their answer to part (e)(iii).

Hence the volume is greater than any cylinder volume (AG)

[3 marks]

(h) Award **A1** for at least one reasonable answer, for example:

the barrel is full of wine when sold
the barrel/stick is constructed with zero thickness
the stick is straight and inflexible
the hole has no diameter
volume of wine is the only important factor in value
aesthetics are not important

Note: Do not accept statements that relate to the barrel having circular cross-section, for example, 'there are no deformities', 'it is perfectly smooth' as these assumptions have already been made with the chosen model.

[1 mark]

[Total 26 marks]

